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A THERMAL PROBLEM OF FRICTION FOR A HALF-SPACE WITH A CRACK

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We investigate the effect of local frictional heating of the surface of a half-space on the stress intensity factors at the vertices of a surface cut.

1. Introduction. Sliding or rolling contact is often accompanied by fatigue failure of the bodies as a result of nucleation and subsequent propagation of surface cracks [1]. The loading on such a contact consists of two parts: mechanical, caused by contact pressure, and thermal, caused by a heat flux due to frictional heat generation. Solutions of plane isothermal contact problems for a half-space with a crack were obtained in [2-6]. We construct a solution of a corresponding thermal problem of friction.

According to investigations [7, 8], the thermal problem of friction is reduced to determination of the temperature field and of the stressed state induced by it in the body when a portion of its boundary surface (the area of contact) is heated by a distributed heat flux with an intensity q proportional to the specific work of friction. This work, with account for the Amonton law for the coupling of tangential and normal stresses [9], is determined as the product of the friction coefficient f, the slip velocity V, and the contact pressure p. In turn, the distribution of the contact pressure is considered to be known and is taken from the solution of the corresponding isothermal contact problem. The contact-pressure distribution most often used is constant or elliptic, in accordance with the Herz formulas [1].

2. Statement of the Problem. Suppose an elastic heat-conducting half-space with an arbitrarily located internal crack (cut) of length $2l$ is heated on a finite portion of its surface of width $2a$ by a linear heat flux of constant intensity:

$$q = \eta f V p \,. \tag{1}$$

The surface of the half-space outside the strip of heating and the sides of the crack are thermally insulated and are free from external effects. We consider the problem within the framework of plane deformation for an established temperature state. We do not take into account possible contact between the sides of the crack.

We refer the half-space to the rectangular coordinate system Oxy so that the x axis is located on the surface and the center of the crack is located at the point O_1 $(0, -h)$ on the y axis, whose positive direction coincides with the external normal to the free surface at the point O. Moreover, we introduce the local coordinate system $O_1x_1y_1$, whose x_1 axis is directed along the line of the cut and forms an angle ω with the Ox axis (Fig. 1). The connection between these coordinate systems gives the relation

$$z_1 = (z + ih) \exp(-i\omega) \,, \tag{2}$$

where $z = x + iy$; $z_1 = x_1 + iy_1$.

The center of the heating area, located a distance d from the y axis, is connected with the rectangular coordinate system $O'x'y'$, where $x' = x + d$, $y' = y$ (Fig. 1).

3. Heat-Conduction Problem. We represent the temperature $T(x, y)$ of the half-space with the cut as the sum of the temperature $T_0(x, y)$ of the solid half-space (the basic temperature field) and the temperature $T^*(x, y)$ perturbed by the presence of the cut.

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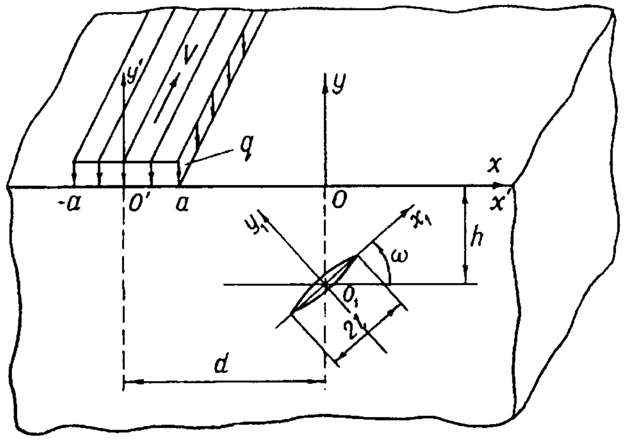

Fig. 1. Scheme of the problem and system of coordinates.

The stationary temperature developed in the solid half-space by heating the portion $|x'| \leq a$ of the surface $y' = 0$ by the linear heat flux (1) is equal to

$$T_0(x, y) = \frac{q}{\pi K}\left\{(x + d - a)\ln\sqrt{(x + d - a)^2 + y^2} - (x + d + a) \times\right.$$

$$\left.\times \ln\sqrt{(x + d + a)^2 + y^2} + y\left[\arctan\left(\frac{x + d - a}{y}\right) - \arctan\left(\frac{x + d + a}{y}\right)\right] + 2a\right\} + c. \tag{3}$$

where c is an arbitrary constant.

The perturbed temperature T^* will be found in the form $T^*(x, y) = \operatorname{Re}\theta(z)$ using the holomorphic function $\theta(z)$, which in turn is associated with the jump $\gamma(x_1)$ in the temperature on the axis of the cut by the relation

$$\theta'(z) = \frac{1}{\pi i}\int_{-l}^{l}\left[\frac{1}{\tau_1 - z} - \frac{1}{\overline{\tau}_1 - z}\right]\gamma'(t_1)\,dt_1, \quad \tau_1 = t_1\exp(i\omega) - ih.$$

Here and below, the prime and the overbar denote a derivative and a conjugate quantity, respectively. The unknown temperature jump $\gamma(x_1)$ is found from the solution of the singular integral equation [10]

$$\frac{1}{\pi}\int_{-l}^{l}\left[\frac{1}{t_1 - x_1} + L(t_1, x_1)\right]\gamma'(t_1)\,dt_1 = F(x_1), \quad |x_1| < l, \tag{4}$$

where

$$L(t_1, x_1) = \operatorname{Re}\left[\exp(i\omega)/(\xi_1 - \overline{\tau}_1)\right], \quad \xi_1 = x_1\exp(i\omega) - ih;$$

$$F(x_1) = -\left.\frac{\partial T_0(x, y)}{\partial y_1}\right|_{y_1=0} = \sin\omega\left.\frac{\partial T_0(x, y)}{\partial x}\right|_{y_1=0} - \cos\omega\left.\frac{\partial T_0(x, y)}{\partial y}\right|_{y_1=0}.$$

Taking into consideration the connection (2), from expression (3) for the basic temperature field $T_0(x, y)$ we find

$$\left.\frac{\partial T_0(x, y)}{\partial x}\right|_{y_1=0} = \frac{q}{\pi K}\left[\ln\sqrt{(x_1\cos\omega + d - a)^2 - (x_1\sin\omega - h)^2} -\right.$$

$$\left.- \ln\sqrt{(x_1\cos\omega + d + a)^2 + (x_1\sin\omega - h)^2}\right],$$

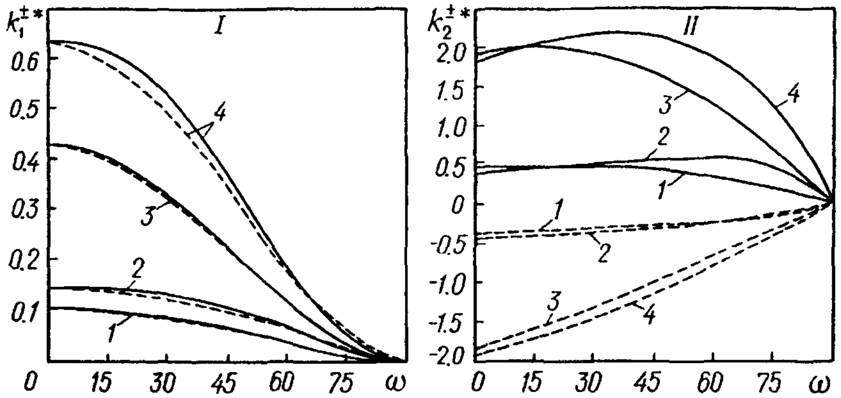

Fig. 2. Dependence of the dimensionless SIF k_1^{+*} (I) and k_2^{+*} (II) (solid curves) and k_1^{-*} (I) and k_2^{-*} (II) (dashed curves) on the angle of orientation of the crack ω in the case of symmetric ($d^* = 0$) heating for $a^* = 0.25$, $l^* = 0.5$ (1), 0.25 and 0.9 (2), 1.5 and 0.5 (3), 1.5 and 0.9 (4).

$$\left.\frac{\partial T_0(x, y)}{\partial y}\right|_{y_1=0} = \frac{q}{\pi K}\left[\arctan\frac{x_1\cos\omega + d - a}{x_1\sin\omega - h} - \arctan\frac{x_1\cos\omega + d + a}{x_1\sin\omega - h}\right].$$

The solution of integral equation (4) must additionally satisfy the condition of continuity of the temperature T^* when going around the contour of the crack:

$$\int_{-l}^{l}\gamma'(t_1)\,dt_1 = 0. \tag{5}$$

4. **Temperature Stresses.** Since the temperature field $T_0(x, y)$ of (4) does not cause stresses in the solid half-space, we find the stresses that are initiated by the perturbed temperature $T^*(x, y)$. The singular integral equation of the corresponding problem of thermoelasticity has the form [10]

$$\int_{-l}^{l}\left[M(t_1, x_1)\,G(t_1) + N(t_1, x_1)\,\overline{G(t_1)}\right]dt_1 = 0, \quad |x_1| < l, \tag{6}$$

where

$$M(t_1, x_1) = \frac{1}{t_1 - x_1} + \frac{\exp(i\omega)}{2}\left\{\frac{1}{\xi_1 - \bar{\tau}_1} + \frac{\exp(-2i\omega)}{\bar{\xi}_1 - \tau_1} + \right.$$
$$\left. + (\bar{\tau}_1 - \tau_1)\left[\frac{1 + \exp(-2i\omega)}{(\xi_1 - \bar{\tau}_1)^2} + \frac{\exp(-2i\omega)(\xi_1 - \tau_1)}{(\bar{\xi}_1 - \tau_1)^3}\right]\right\};$$

$$N(t_1, x_1) = \frac{\exp(i\omega)}{2}\left[\frac{\tau_1 - \bar{\tau}_1}{(\xi_1 - \bar{\tau}_1)^2} + \frac{1}{\bar{\xi}_1 - \tau_1} - \frac{\exp(-2i\omega)(\xi_1 - \tau_1)}{(\bar{\xi}_1 - \tau_1)^2}\right];$$

$$G(t_1) = g(t_1) + i\beta\gamma(t);\quad \beta = \alpha\mu(1 + \nu)/(1 - \nu);$$

$g(t_1)$ is the jump of normal (in the direction of the y_1 axis) displacements when passing over the line of the crack. Uniqueness of these displacements in passing around the contour of the crack is ensured by satisfaction of the condition

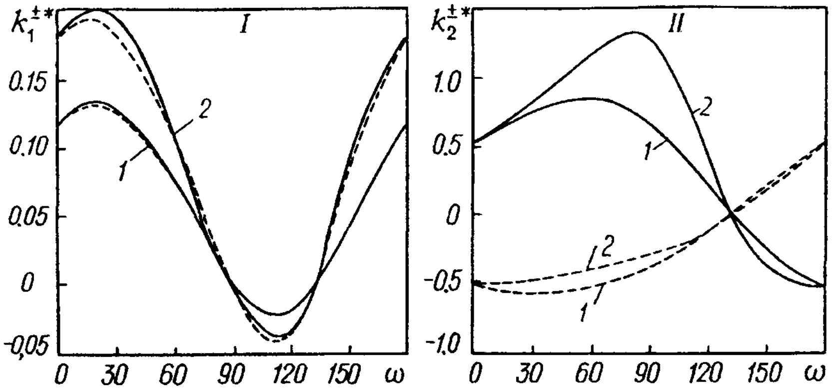

Fig. 3. Dependence of the dimensionless SIF k_1^{+*} (I) and k_2^{+*} (II) (solid curves) and k_1^{-*} (I) and k_2^{-*} (II) (dashed curves) on the angle of orientation of the crack ω in the case of asymmetric ($d^* = 1$) heating for $a^* = 0.25$ and $l^* = 0.5$ (1), 0.9 (2).

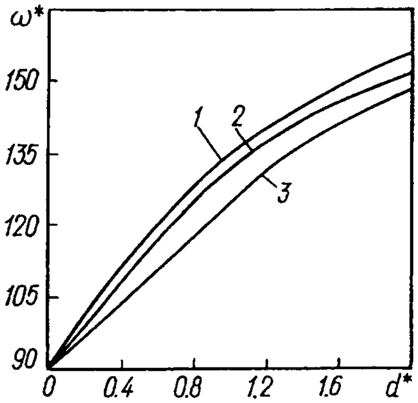

Fig. 4. Dependence of the angle of orientation of the crack ω^* at which $k_i^{\pm *} = 0$, $i = 1, 2$, on the parameter d^* for $l^* = 0.5$ and $a^* = 0.25$ (1), 0.5 (2), 1 (3).

$$\int_{-l}^{l} G(t_1)\, dt_1 = -i\beta \int_{-l}^{l} t_1 \gamma'(t_1)\, dt_1 . \tag{7}$$

5. Numerical Analysis. The algorithm of the solution of the problem considered consists in the following: from integral equation (4), with condition (5) being satisfied, we find the derivative of the temperature jump $\gamma'(t_1)$, using which we solve the system of integral equations (6) and (7) for the function $G(t_1)$; using the formula

$$k_1^{\pm} - ik_2^{\pm} = \mp \lim_{t_1 \to \pm l} \sqrt{2\pi\, |t_1 \mp l|}\; G(t_1)$$

we determine the stress intensity factors (SIF) $k_i^{\pm}$, $i = 1, 2$ at the vertices $x_1 = \pm l$ of the crack.

The solution of the singular integral equations of the first kind (4), (6) with a singular Cauchy kernel in the class of functions of index 1 (i.e., ones possessing a root singularity at the ends $\pm l$ of the integration interval) was obtained numerically by the method of mechanical quadratures [11]. The dimensionless half-width of the region of frictional heating $a^* = a/h$, displacement of the center of this region $d^* = d/h$, and half-length of the cut $l^* = l/h$ are the independent input parameters of the problem. To attain a relative accuracy of the calculations of 1%, not more than 20 collocation points were needed. Results for the dimensionless SIF $k_i^{\pm *} = k_i^{\pm} 2\pi K / q\beta l\sqrt{\pi l}$, $i = 1, 2$, are presented in Figs. 2-5. Here, the solid curves correspond to the SIF k_i^{+*}, $i = 1,2$, at the vertex of the cut $x_1 = l$, and the dashed curves correspond to the SIF k_i^{-*}, $i = 1, 2$, at the vertex $x_1 = -l$.

When the crack is located symmetrically with respect to the section of heating ($d^* = 0$), the SIF $k_1^{\pm *}$ acquire their greatest value at $\omega = 0$ (the crack is parallel to the surface of the half-space) irrespective of the width of the

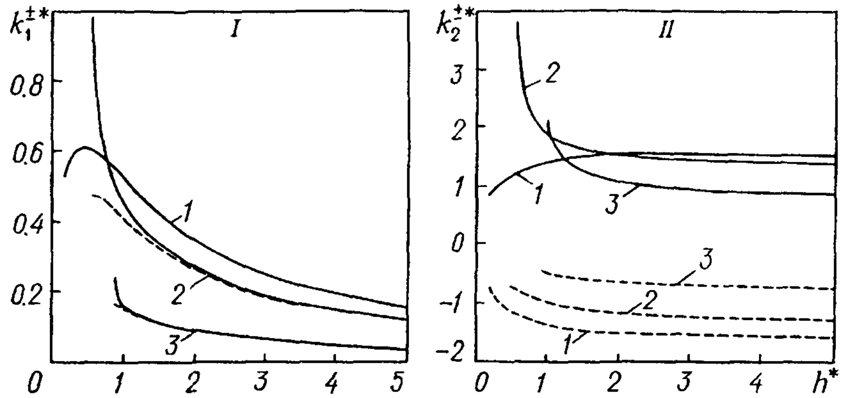

Fig. 5. Dependence of the dimensionless SIF k_1^{+*} (I) and k_2^{+*} (II) (solid curves) and k_1^{-*} (I) and k_2^{-*} (II) (dashed curves) on the parameter h^* in the case of symmetric ($d^* = 0$) heating for $a^* = 1$ and $\omega = 0$ (1), 30° (2), 60° (3).

area of heating or the length of the cut (Fig. 2, I). The value of the angle ω at which the SIF k_2^{+*}, which characterizes the intensity of tangential stresses at the vertex of the cut closer to the surface of the half-space, attains its maximum depends substantially on the width of the region of heating (Fig. 2, II). Thus, when the parameter l^* is increased from 0.1 to 0.9, this angle increases from 0 to 70° for $a^* = 0.25$ and from 0 to 45° for $a^* = 1.5$. At $\omega = 90°$ the SIF $k_i^{\pm *}$, $i = 1, 2$, are equal to zero, since then the thermally insulated crack does not perturb the temperature field.

Asymmetry in the mutual arrangement of the region of frictional heating and the crack leads to an increase in the angle of orientation of the crack ω at which the SIF acquires its maximum value (Fig. 3, I). The maximum of $k_2^{\pm *}$ is attained for $60° < \omega < 90°$ from 0 to 30° (Fig. 3, II).

Figure 3 also contains an important result: for fixed parameters a^* and d^*, irrespective of the dimensionless length of the cut l^*, there is an angle of orientation of the crack $\omega = \omega^*$ at which $k_1^{\pm *} = k_2^{\pm *} = 0$ simultaneously. For a prescribed width of the zone of heating a^*, irrespective of the length of the cut l^*, an increase in the parameter d^* from 0 to 2 leads to an increase in ω^* from 90 to 160°, respectively (Fig. 4). For a fixed displacement d^* the value of the angle ω^* is larger, the narrower the strip of frictional heating.

The effect of the distance between the center of the cut and the surface of the half-space (the parameter $h^* = h/l$) on the SIF in the case of symmetric ($d^* = 0$) heating is shown in Fig. 5. Approach of the cut to the free surface of the half-space causes a monotonic increase in the SIF. However, very close to this surface a sharp decrease in $k_i^{\pm *}$, $i = 1, 2$, is observed for $\omega = 0$. A similar behavior of the SIF was noted earlier in [12] when a finite portion of the surface of the half-space was maintained at a constant temperature. But in the problem considered a finite portion is heated by a constant heat flux.

6. **Conclusions.** It is found that for a prescribed power of frictional heat generation there is a unique correspondence between the angle of orientation of the crack $\omega = \omega^*$ at which no thermally stressed state develops in the half-space and the parameter d^* that characterizes the distance from the center of the section of heating to the center of the cut. Physically this can be explained by the fact that the crack is positioned in this case in such a way that its plane coincides with the isothermal surface. The effect of the free surface of the half-space on the SIF at the vertices of the cut is revealed primarily in an increase in the SIF when the crack approaches the surface of the half-space and in a sharp decrease in the SIF for a surface cut that is parallel to the boundary of the half-space.

NOTATION

a, half-width of the strip of heating; d, distance from the center of the region of heating to the y axis; f, friction coefficient; h, distance from the center of the crack to the surface of the half-space; K, thermal conductivity; l, half-length of the crack; p, contact pressure; q, heat-flux intensity; T, temperature; V, slip velocity; $a^* = a/h$, $d^* = d/h$, $h^* = h/l$, $l^* = l/h$, dimensionless geometric parameters; α, coefficient of linear thermal expansion; η, coefficient of separation of heat fluxes; μ, shear modulus; ν, Poisson coefficient; ω, angle of orientation of the crack.

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